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Solving String Field Equations: New Uses for Old Tools

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Abstract: This is the contents of a talk by O. L. presented at the 35th International Symposium Ahrenshoop in Berlin, Germany, 26–30 August 2002. It is argued that the (NS-sector) superstring field equations are integrable, i. e. their solutions are obtainable from linear equations. We adapt the 25-year-old solution-generating "dressing" method and reduce the construction of nonperturbative superstring configurations to a specific cohomology problem. The application to vacuum superstring field theory is outlined.

1 Zero-curvature and linear equations (old tools)

The flatness of a gauge connection A,[1]

The situation changes when a linear parametric dependence on some $\lambda \in \mathbb{C}P^1$ is introduced,

The extended linear system associated with (**??**) is

2 Single-pole ansatz

The simplest non-constant meromorphic function possesses a single pole. The corresponding ansatz,[2]

It turns out that all information resides in eqs. (**??**) and (**??**). Since their left hand sides are independent of λ, the poles at $\lambda{=}\mu$ and $\lambda{=}-1/\bar{\mu}$ of their right hand sides must be removable. Putting to zero the residues in (**??**) yields algebraic relations,

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[1]We suppress the spacetime coordinate dependence throughout.
[2]This is an essential building block in the "dressing method", a solution-generating technique invented by [3, 4] and developed by [5].

3 String fields (new uses)

The complete structure presented so far carries over almost verbatim to string field theory. Instead of being Lie-group or Lie-algebra valued, our objects now are interpreted as string fields and are to be multiplied via Witten's star product [6] (which we suppress). More precisely, the unextended situation (**??**) and (**??**) corresponds to the cubic open bosonic string [6], with $d \mapsto Q$ (the BRST operator) and A having ghost number one. The linear system $(Q{+}A)\Psi = 0$ yields only trivial solutions to the string field equation of motion, $QA{+}A^2 = 0$.

Surprisingly, the extended case (**??**) and (**??**) can be mapped onto the NS sector of cubic open *superstring* field theory [7], by $d \mapsto Q$ and $\widetilde{\mathrm{d}} \mapsto \eta_0$. Here, η_0 denotes the graded commutator action of the zero mode of η, which emerges from bosonizing the worldsheet supersymmetry ghosts via $\gamma = \eta\,\mathrm{e}^{\varphi}$ and $\beta = \mathrm{e}^{-\varphi}\partial\xi$. The NS string field A carries ghost number one and picture number zero and lives in the "large Hilbert space" (including ξ_0). Consequentially, its equations of motion [8, 9] are

It is known that $Q(\lambda) \equiv \eta_0{+}\lambda Q$ has zero cohomology in the large Hilbert space. Therefore, we can get all solutions to (**??**) from a "linear superstring system" [11],

4 Ghost-picture modification

Our "master equation" (**??**) has a flaw: It is inhomogeneous in picture number because η_0 lowers the picture by one unit. Therefore, any solution Ψ is an *infinite sum* over all picture sectors (requiring an extension of standard superstring field theory), unless we modify our equation by introducing a picture-raising multiplier,

5 Shifting the background

We may view solving (**??**) as "dressing" the vacuum solution,

6 Tachyon vacuum superstring fields

Of particular interest is the structure of (super)string field theory around the (NS) tachyon vacuum. Let us suppose the latter can be reached as A_1 within our ansatz. It happens to be useful to redefine the fluctuations around A_1 by

Assuming [14] that the vacuum string fields describing D-branes factorize,

Let us briefly expand on the last remark. From

7 Outlook

The "linearization" of the superstring field equations provides us with a new window to nonperturbative superstring physics. The ideas presented here are just the beginning. Among future developments one may consider

- the construction of nontrivial classical string fields ($\mathcal{A}\neq 0$)
- the precise relation with noncommutative solitons (in the Moyal basis)
- the generalization to the multi-pole ansatz for $\Psi(\lambda)$
- the interpretation of solutions (as D-branes?) and their moduli μ
- the computation of energy densities, tensions, etc. for a given solution
- the analysis of fluctuations around constructed classical configurations
- the extension to the Ramond sector

First steps have been made in [18].

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References

[1] A. N. Leznov and M. A. Mukhtarov, J. Math. Phys. **28** (1987) 2574.

[2] C. N. Yang, Phys. Rev. Lett. **38** (1977) 1377.

[3] V. E. Zakharov and A. V. Mikhailov, Sov. Phys. JETP **47** (1978) 1017 [Zh. Eksp. Teor. Fiz. **74** (1978) 1953].

[4] V. E. Zakharov and A. B. Shabat, Funct. Anal. Appl. **13** (1979) 166.

[5] P. Forgacs, Z. Horvath and L. Palla, Nucl. Phys. B **229** (1983) 77.

[6] E. Witten, Nucl. Phys. B **268** (1986) 253.

[7] E. Witten, Nucl. Phys. B **276** (1986) 291.

[8] C. R. Preitschopf, C. B. Thorn and S. Yost, Nucl. Phys. B **337** (1990) 363.

[9] I. Ya. Aref'eva, P. B. Medvedev and A. P. Zubarev, Nucl. Phys. B **341** (1990) 464.

[10] N. Berkovits, Nucl. Phys. B **450** (1995) 90, Erratum ibid. B **459** (1996) 439 [hep-th/9503099].

[11] O. Lechtenfeld, A. D. Popov and S. Uhlmann, Nucl. Phys. B **637** (2002) 119 [hep-th/0204155].

[12] D. Gaiotto, L. Rastelli, A. Sen and B. Zwiebach, hep-th/0111129.

[13] K. Ohmori, Nucl. Phys. B **648** (2003) 94 [hep-th/0208009].

[14] L. Rastelli, A. Sen and B. Zwiebach, Adv. Theor. Math. Phys. **5** (2002) 353 [hep-th/0012251].

[15] M. Marino and R. Schiappa, hep-th/0112231.

[16] K. Ohmori, JHEP **0204** (2002) 059 [hep-th/0204138].

[17] I. Ya. Aref'eva, A. S. Koshelev, D. M. Belov and P. B. Medvedev, Nucl. Phys. B **638** (2002) 3 [hep-th/0011117];
I. Ya. Aref'eva, D. M. Belov, A. S. Koshelev and P. B. Medvedev, Nucl. Phys. B **638** (2002) 21 [hep-th/0011117];
I. Ya. Aref'eva, A. A. Giryavets and P. B. Medvedev, Phys. Lett. B **532** (2002) 291 [hep-th/0112214];
I. Ya. Aref'eva, D. M. Belov and A. A. Giryavets, JHEP **0209** (2002) 050 [hep-th/0201197].

[18] A. Kling, O. Lechtenfeld, A. D. Popov and S. Uhlmann, Phys. Lett. B **551** (2003) 193 [hep-th/0209186].